

International Journal of Modern Physics A
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ON THE IMPLICATIONS OF A DILATON IN GAUGE THEORY

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Some recent work on the implications of a dilaton in 4d gauge theories are revisited. In part I of this paper we see how an effective dilaton coupling to gauge kinetic term provides a simple attractive mechanism to generate confinement. In particular, we put emphasis on the derivation of confining analytical solutions and look into the problem how dilaton degrees of freedom modify Coulom potential and when a confining phase occurs. In part II, we solve the semi-relativistic wave equation, for Dick interquark potential using the Shifted l-expansion technique (SLET) in the heavy quarkonium sector. The results of this phenomenological analysis proves that these effective theories can be relevant to model quark confinement and may shed some light on confinement mechanism.

Keywords: dilaton, confinement, quark potential

1. Introduction

Despite enormous amount of work performed over more than thirty years, particularly in lattice simulations of QCD, full Understanding of the QCD vacuum structure and color confinement mechanism are still lacking. Indeed, direct derivation of confinement from first principles remain still elusive and there is no totally convincing proposal about its generating mechanism. However, on the other hand, we known that the vacuum topological structure of theories with dilaton fields is drastically changed compared to the non dilatonic ones ¹. Therefore much about confinement might be learned from such theories, particularly string inspired ones. The presence of fundamental scalars with direct coupling to gauge curvature terms in string theories offers a challenge with attractive implications in four-dimensional gauge theories. ^a Besides, since color confinement can be signaled through the behavior of the interaction potential at large distances. In this context, it was suggested in ³ that an effective coupling of a massive dilaton to the 4-dimensional gauge fields may provide an interesting mechanism which accommodates both the Coulomb and confining phases. The derivation performed in ^{3,4} suggest a new scenario to generate color confinement. This scenario may be considered as a challenge

^aThe dilaton is an hypothetical scalar particle predicted by string theory and Kaluza-Klein type theories. Its expectation value probes the strength of the gauge coupling ².

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to the mechanism based on monopole condensation.

The outline of this paper is as follows. In the part I, we describe the influence of the dilaton on a low energy gauge theory and look into the problem how dilatonic degrees of freedom modifies Coulomb potential and how transition to a confining phase occurs. Then, we review several recent work by presenting the corresponding effective coupling functions used. We briefly comment on the analytic solutions of the field equations and their confinement features. Part II is devoted to phenomenological investigations. We study Dick interquark potential in the heavy quarkonium systems using SLET technique. and summarize the results obtained from this analysis. Finally, we draw our general conclusion.

2. The low energy effective theory

The imprint of dilaton on a 4d effective nonabelian gauge theory is described by a Lagrangian density:

$$\mathcal{L}(\phi, A) = -\frac{1}{4F(\phi)}G_{\mu\nu}^a G_a^{\mu\nu} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + J_a^\mu A_\mu^a \quad (1)$$

where ϕ is the dilaton field and $G^{\mu\nu}$ is the standard field strength tensor of the theory. $V(\phi)$ denotes the non perturbative dilaton potential and $F(\phi)$ represents the coupling function depending on ϕ . Several forms of $F(\phi)$ have been proposed in literature. The most popular one $F(\phi) = e^{-k\frac{\phi}{f}}$ occurred in string theory and Kaluza-Klein theories ².

Analysis of the problem of Coulomb gauge theory augmented with dilaton degrees of freedom in (1) performed as follows:

First, we consider a point like static Coulomb source defined in the rest frame by the current:

$$J_a^\mu = g\delta(r)C_a\nu_0^\mu = \rho_a\eta_0^\mu \quad (2)$$

where C_a is the expectation value of $SU(N_c)$ generator. The field equations emerging from the static configuration (2) are given by:

$$[D_\mu, F^{-1}(\phi)G^{\mu\nu}] = J^\nu \quad (3)$$

and

$$\partial_\mu\partial^\mu\phi = -\frac{\partial V(\phi)}{\partial\phi} - \frac{1}{4}\frac{\partial F^{-1}(\phi)}{\partial\phi}G_a^{\mu\nu}G_a^{\nu\mu} \quad (4)$$

By setting $G_a^{0i} = E^i\chi_a = -\nabla^i\Phi_a$, and after some algebra, we derive the chromo-electric field:

$$E_a = \frac{Q_{eff}^a(r)}{r^2} \quad (5)$$

where the effective charge is

$$Q_{eff}^a(r) = \left(g\frac{C_a}{4\pi}\right)F(\phi(r))$$

Eq(5) shows that it is the running of the effective charge that makes the potential stronger than the Coulomb potential. In other words, the Coulomb spectrum is recovered if the effective charge did not run. Thereby the interquark potential reads as ⁴,

$$U(r) = 2\tilde{\alpha}_s \int \frac{F(\phi(r))}{r^2} dr \quad (6)$$

with $\alpha_s = \frac{g^2}{4\pi}$ and $\tilde{\alpha} = \frac{\alpha_s}{8\pi} \left(\frac{N_c-1}{2N_c} \right)$

The formula in Eq. (6) is remarkable since it provides a direct relation between the interquark potential and the coupling function $F(\phi(r))$. Moreover, it shows that existence of confining phases in this effective theory is subject to the following condition,

$$\lim_{r \rightarrow \infty} r F^{-1}(\phi(r)) = \text{finite} \quad (7)$$

At this stage, the main objective is to solve the field equations of motion (3) and (4) and determine analytically $\phi(r)$ and $\Phi_a(r)$. For this, $F(\phi)$ and $V(\phi)$ have to be fixed. In the sequel the dilaton potential is set to $V(\phi) = \frac{1}{2}m^2\phi$. Below, we will briefly describe the main features of three recent models and present their solutions.

2.1. Dick Model

In this effective theory, Dick used the form: $\frac{1}{F(\phi)} = \frac{\phi^2}{f^2 + \beta \phi^2}$ where f represents a coupling scale characterizing the strength of the scalar-gluon coupling and β is a parameter in the range $[0, 1]$. He derived the radial dependence of the dilaton field and the interquark potential (up to a color factor) ³:

$$\phi(r) = \pm \frac{1}{r} \sqrt{\frac{k}{m} + (y_0^2 - \frac{k}{m}) \exp(-2mr)}$$

$$V(r) = [\frac{\beta g^2}{4\pi r} - gf \sqrt{\frac{N_c}{2(N_c-1)}} \ln[e^{2mr} - 1 + \frac{m}{k} y_0^2]$$

With the abbreviation: $k^2 = \frac{\alpha_s f^2}{8\pi} \frac{N_c-1}{N_c}$

Remarkably the potential $V(r)$ comes with the required behavior: a first term which accommodates the Coulomb interaction at short distances and a second term linearly increasing in the asymptotic regime with a string tension ^b

$\sigma \sim g m f$ which depends on the dilatonic degrees of freedom m, f .

^bIn the massless case, $V(\phi) = 0$, solutions of the field equations reduced to: $\phi(r) = \pm \left(\frac{gf}{2\pi} \right) \sqrt{\frac{N_c-1}{N_c}} r^{-\frac{1}{2}}$, $V(r) = \frac{g^2 \beta (N_c-1)}{8\pi r N_c} - \frac{fg}{2} \sqrt{\frac{N_c-1}{N_c}} r$

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2.2. *Cornwall-Soni Model*

In this model, the glueballs are represented by a massive scalar field ϕ and couple in a non minimal way to gluons, through $\frac{1}{F(\phi)} = \frac{\phi}{f}$ ⁵. Cornwall-Soni were the first to motivate such term as a low energy correction to effective models of QCD

Analytical Solutions were found for $r \rightarrow \infty$ ⁶,

$$\phi(r) = \left[\frac{\alpha_s f (N_c - 1)}{16 \pi m^2 N_c} \right]^{\frac{1}{3}} r^{-\frac{4}{3}}$$

$$V(r) = -3g \frac{N_c - 1}{2 N_c} \left[\frac{g f^2 N_c m^2}{\pi (N_c - 1)} \right]^{\frac{1}{3}} r^{\frac{1}{3}}$$

These formulas show that at large distances, confinement is probed through an interaction potential proportional to $r^{1/3}$ and considered by the authors as non perturbative correction to the Coulomb phase.

2.3. *Chabab-Sanhaji Model*

The main aim in this work was to construct a low energy effective field theory from which some popular phenomenological potentials may emerge. To this end, we proposed the coupling function $F(\phi) = \left(1 - \beta \frac{\phi^2}{f^2}\right)^{-n}$ ⁷.

By substituting $F(\phi)$ Eq. (3, 4), the field equations were found too complicated to integrate analytically. Fortunatly, since the focus is on the long range behavior of the dilaton field and on how it modifies the Coulomb phase, the analysis is restricted to the infrared region. Thus, the asymptotic solutions are found to be,

$$\phi = \left[\frac{f^2}{\beta} - \left(\frac{\beta}{f^2} \right)^{\frac{-n}{n+1}} \left(\frac{2n\alpha_s}{m^2} \right)^{\frac{1}{n+1}} \left(\frac{1}{r} \right)^{\frac{4}{n+1}} \right]$$

and the chromo-electric potential:

$$\Phi_a(r) = -\frac{gC_a}{4\pi} \left(\frac{2n\alpha_s}{m^2 f^2} \right)^{\frac{-4n}{n+1}} \frac{n+1}{3n-1} r^{\left(\frac{3n-1}{n+1} \right)}$$

We see that the occurrence of confinement depends on the parametre n and our effective theory can serve to model quark confinement when $n \in \left[\frac{1}{3}, 1 \right]$.

On the other hand, going back to the objective of this study: by selecting specific values of n , we reproduced the following known interquark potentials

- $n = 1 \Rightarrow$ linear term of Cornwall potential⁸.
- $n = 11/29 \Rightarrow$ Martin's potential⁹.
- $n = 3/5 \Rightarrow$ Song-Lin, or Motyka-Zalewski' potential¹⁰.
- $n = 5/9 \Rightarrow$ Turin potential¹¹.

Therefore, these quark potentials, which gained credibility only through confrontation to the hadron spectrum, are now supplied with a theoretical framework since they can be derived from a low energy effective theory.

3. Phenomenological Analysis

Our aim in this part of the review paper is to dedicate more efforts to understand the new confinement mechanism suggested above through phenomenological investigation of Dick potential in the heavy meson sector. This study will be addressed as in ¹² where the shifted- l expansion technique is used (SLET) where l is the angular momentum. This method provides a powerful analytic technique for determining the bound states of the semi-relativistic wave equation consisting of two quarks of masses m_1 , m_2 and total binding meson energy M in any spherically symmetric potential. It is rapidly converging and handles highly excited states which pose problems for variational methods ¹³. Moreover, relativistic corrections are included in a consistent way.

Dick interquark potential reads,

$$V_D(r) = -\frac{4}{3} \frac{\alpha_s}{r} + \frac{4}{3} g f \sqrt{\frac{N_c}{2(N_c - 1)}} \ln[\exp(2mr) - 1] \quad (8)$$

The SLET technique used to obtain results from the theory requires us to specify several inputs: m_c , m_b , m , f and α_s . In our numerical analysis, we set the charm and bottom quark masses to the values $m_c = 1.89$ GeV and $m_b = 5.19$ GeV. For the QCD coupling constant, in contrast to the Lattice potentials which use the same effective coupling in the description of heavy quarkonium, we take into account the running of α_s ,

$$\alpha_s(\lambda) = \frac{\alpha_s(m_z)}{1 - (11 - \frac{2}{3}n_f)[\alpha_s(m_z)/2\pi]\ln(m_z/\lambda)}, \quad (9)$$

where the renormalization scale is fixed to $\lambda = 2\mu$, with μ is the reduced mass,

$$\mu = \frac{m_1 m_2}{m_1 + m_2}, \quad (10)$$

Thus, combination of the leading order formula (9) and the world experimental value $\alpha_s(m_z) = 0.12$ yields,

$$\alpha_s(charmonium) = 0.31, \quad \alpha_s(bottomonium) = 0.20, \quad (11)$$

while $\alpha_s = 0.22$ for the $b\bar{c}$ quarkonia. On the other hand, the interquark potential parameters m and f are treated as being free in our analysis and are obtained by fitting the spin-averaged $c\bar{c}$ and $b\bar{b}$ bound states. An excellent fit with the available

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experimental data can be seen to emerge when the following values are assigned ^c,

$$m = 57 \text{ MeV} \quad gf \sqrt{\frac{N_c}{2(N_c - 1)}} = 430 \text{ MeV}. \quad (12)$$

Table 1. Calculated mass spectra (in units of GeV) $M_{n\ell}$ of $c\bar{c}$ boundstates from Dick interquark potential

State, $n\ell$	$M_{n\ell}$, SLET	$M_{n\ell}$, Exp.	State, $n\ell$	$M_{n\ell}$, SLET	$M_{n\ell}$, Exp.
1S	3.073	3.068	1P	3.546	3.525
2S	3.662	3.663	2P	3.871	-
3S	4.027	4.028	1D	3.787	3.788

Table 2. Calculated mass spectra (in units of GeV) $M_{n\ell}$ of $b\bar{b}$ from Dick interquark potential

State, $n\ell$	$M_{n\ell}$, SLET	$M_{n\ell}$, Exp.	State, $n\ell$	$M_{n\ell}$, SLET	$M_{n\ell}$, Exp.
1S	9.450	9.446	1P	9.903	9.900
2S	10.014	10.013	2P	10.227	10.260
3S	10.299	10.348	1D	10.129	-

Table 3. Calculated mass spectra (in units of GeV) $M_{n\ell}$ of $b\bar{c}$ boundstates from Dick potential

State, $n\ell$	$M_{n\ell}$, SLET	$M_{n\ell}$, Exp.	State, $n\ell$	$M_{n\ell}$, SLET	$M_{n\ell}$, Exp.
1S	6.322	$6.276 \pm 0.004 \pm 0.0027$	1P	6.767	-
2S	6.876	-	2P	7.072	-
3S	7.181	-	1D	6.994	-

Tables (1,2) list the results of the analysis for the spin-averaged energy levels of interest. In all cases, where comparison with experiment is possible, agreement is generally very good. Next step, to check the consistency of our predictions, we estimate the bound states energies of the $b\bar{c}$ quarkonia. These states are expected to be produced at LHC and Tevatron. Moreover, they should provide an excellent test to discriminate between various techniques used to probe nonperturbative properties of hadrons. In table 3 we show our calculated spectrum. The estimate of the mass of the lowest pseudoscalar S-state of the B_c spectra is close to the experimental value reported by CDF and D0 collaborations¹⁴. As to the higher states masses, they compare favorably with other predictions based on QCD sum-rules^{15,16} or potential models¹⁷⁻²³. In conclusion, Dick interquark potential (08) is tested successfully to fit the spin-averaged quarkonium spectrum. In view of these results, it

^cif the standard value for the string tension 0.18 GeV^2 is used, the dilaton mass will be shifted to a value about 158 MeV .

is quite encouraging to pursue phenomenological application of $V_D(r)$ and other quark potentials emerging from such low effective gauge theory with dilaton.

4. General conclusion

We revisited some of the most recent work on confinement in 4d non abelian gauge theories with a massive scalar field (dilaton) and effective coupling functions to gauge fields. Analytical solutions have been found with confinement feature at large distances. Thus, These low energy effective theories can serve well to model quark confinement. Moreover, by using Dick interquark potential in the heavy quarkonium sector, we showed that phenomenological investigation of the confinement generating mechanism suggested by these models is more than justified. Indeed, the obtained results for charmonium and bottomonium fit well experimental data when the dilaton mass is given a value about 57 MeV. Also, for B_c system, we found that the S-state energy level is close to the value reported by CDF collaboration, while those of excited states agree favorably with predictions of other theoretical studies. On the other hand, as a by-product, this analysis allows a test to the physics beyond the standard model in relation to hadron spectroscopy. Indeed, the estimate of the dilaton mass lies in the range of values proposed in ^{24,25}, which may shed some light on the search of the dilaton since the possibility to identify this hypothetical particle to a fundamental scalar invisible to present day experiments should not be ruled out ^{26,27}.

Acknowledgments

This Work is partially supported by the Morocco research program PROTARS III, under contract D16/04.

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